CHOQUET INTEGRAL WITH RESPECT TO THE
COMPOSED FUZZY MEASURE OF COMPLETED
L-MEASURE AND DELTA-MEASURE
結合完全 L-測度與 Delta-測度之複合模糊測
度之 CHOQUET 積分
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摘要

如眾所周知，傳統的 $\lambda$-測度和 $P$-測度，均僅具有唯一公式解。為了改善此一缺失與限制，我們先後分別提出具有無窮多解的 $L$-測度與 $\delta$-測度，但 $L$-測度並不是可加性測度，而且 $\delta$-測度之多值測度解之範圍比 $L$-測度之多值測度解之範圍小很多。

由於上述兩種測度各有其優劣，為了結合優點，避除上述缺點，結合 $L$-測度與半個 $\delta$-測度，我們發表複合模糊測度模式。本研究更上一層樓，提出更為改善的模糊測度，結合 $L$-測度與全部的 $\delta$-測度，稱之為「結合完全 $L$-測度與 Delta-測度之複合模糊測度模式」，記做 $L(C\delta)$。

研究結果顯示：結合完全 $L$-測度與 Delta-測度之複合模糊測度模式，$L(C\delta)$，在基於 $\gamma$-密度函數之 Choquet 模糊積分迴歸模式之預測效力優於其他預測模式。

關鍵字：$\lambda$-測度，$P$-測度，$L$-測度，$\delta$-測度，$L(\delta)$-模糊測度，$\gamma$-密度函數，Choquet 模糊積分，$L(C\delta)$ 完全模糊測度
ABSTRACT

In this dissertation, a composed fuzzy measure of completed $L$-measure and $\delta$-measure, denoted $L_{C\delta}$-measure, is proposed. This new measure is proved that it is of closed form with infinitely many solutions, and can be considered as an extension of the three well known measures, additive measure, $\lambda$-measure and $P$-measure, respectively. Furthermore, it is a completed multivalent fuzzy measure, and not only including the smallest fuzzy measure, $P$-measure, but also attaining to the largest fuzzy measure, $B$-measure. It has more infinitely many fuzzy measure solutions than $L$-measure, $\delta$-measure and the composed fuzzy measure of $L$-measure, and $\delta$-measure. By using 5-fold cross-validation MSE, a real data experiment is conducted for comparing the performances of a multiple linear regression model, a ridge regression model, and the Choquet integral regression model with respect to $P$-measure, $\lambda$-measure, $\delta$-measure, $L$-measure, $L(\delta)$-measure, $L(C)$-measure and $L(C\delta)$-measure, respectively.

The result shows that the Choquet integral regression models with respect to the proposed $L(C\delta)$-measure outperforms other forecasting models.

keywords: $\lambda$-measure, $P$-measure, $L$-measure, $\delta$-measure, $L(\delta)$-measure, $\gamma$-Density Function, Choquet integral, $L(C\delta)$-measure
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CHAPTER I THE PROBLEM

1-1 Introduction

When there are interactions among independent variables, traditional multiple linear regression models do not perform well enough. The traditional improved methods exploited ridge regression models (Browne, 2000; Hoerl, Kenard, & Baldwin, 1975). Recently, the Choquet integral regression models (Liu, 2009; Liu, Tu, Huang, & Chen, 2008; Liu, Tu, Lin, & Chen, 2008; Liu, Wu, Jheng, & Sheu, 2009; Liu, Chen, Chen, & Jheng, 2007) based on some univalent or multivalent fuzzy measures (Chen, Jheng, Yao, & Liu, 2008; Liu, 2009; Liu, Chen, Jheng, & Chien, 2009; Liu, Chen, Wu, & Sheu, 2009; Liu, Jheng, Lin, & Chen, 2007; Liu, Lin, & Weng, 2007; Liu, Lin, Chang, & Weng, 2007; Liu, Tu, Chen, & Weng, 2008; Liu, Wu, Jheng, & Sheu, 2009; Sugeno, 1974; Wang & Klir, 1992; Wang & Klir, 2009; Zadeh, 1978) were used to improve this situation.

The fuzzy measures, $\lambda$-measure (Sugeno, 1974; Wang & Klir, 1992; Wang & Klir, 2009) and $P$-measure (Zadeh, 1978) have only one formulaic solution of fuzzy measure, the former is not a closed form, and the latter is not sensitive enough. Two multivalent fuzzy measures with infinitely many solutions were proposed by our previous works, called $L$-measure (Liu, 2009; Liu, Chen, Jheng, & Chien, 2009) and $\delta$-measure (Liu, Wu, Jheng, & Sheu, 2009), but $L$-measure does not include the additive measure and $\delta$-measure has no multiple measure solutions as $L$-measure. Due to the above drawbacks, an improved fuzzy measure composed of $L$-measure and $\delta$-measure, denoted $L_\delta$-measure, was proposed by our other previous work (Liu, Chen, Wu, & Sheu, 2009; Liu, Wu, Chen, Tsai, Jheng, & Sheu, 2009; Liu, Wu, Chen, & Jheng, 2010).

1-2 Objectives of the Study

The main objectives of this study were as follows:

1. A further improved fuzzy measure composed of completed $L$-measure and
δ-measure, denoted $L_{C\delta}$-measure is proposed. This new fuzzy measure is a completed multivalent fuzzy measure, and has infinitely fuzzy measure solutions than three multivalent fuzzy measures: $L_\delta$-measure, L-measure, and δ-measure, respectively.

2. For evaluating the Choquet integral regression models with our proposed fuzzy measure and other different ones, a real data experiment by using a 5-fold cross-validation mean square error (MSE) is conducted. The performances of Choquet integral regression models based on $L_{C\delta}$-measure, $L_\delta$-measure, L-measure, δ-measure, λ-measure, and P-measure, respectively, a ridge regression model, and a multiple linear regression model are compared.
CHAPTER II  LITERATURE REVIEW

2-1  Multiple Linear Regression and Ridge Regression

Let \( Y = X\beta + \varepsilon, \varepsilon \sim N(0, \sigma^2 I_n) \) be a multiple linear model, \( \hat{\beta} = (X'X)^{-1} X'Y \) be the estimated regression coefficient vector, and \( \hat{\beta}_k = (X'X + kI_n)^{-1} X'Y \) be the estimated ridge regression coefficient vector, Hoerl, Kenard and Baldwin (1975) suggested

\[
\hat{k} = \frac{n\sigma^2}{\hat{\beta}'\hat{\beta}}
\]  

(2.1)

2-2  Fuzzy Measures

The two well known fuzzy measures, the \( \lambda \)-measure proposed by Sugeno in 1974, and \( P \)-measure proposed by Zadah in 1978, are concisely introduced as follows.

2-2-1 Axioms of Fuzzy Measures

Definition 2.1  fuzzy measure

A fuzzy measure (Sugeno, 1974; Wang & Klir, 1992; Wang & Klir, 2009) \( \mu \) on a finite set \( X \) is a set function \( \mu: 2^X \rightarrow [0,1] \) satisfying the following axioms:

\[
(1) \mu(\emptyset) = 0, \mu(X) = 1 \quad \text{(Boundary conditions)}
\]

(2.2)

\[
(2) A \subseteq B \Rightarrow \mu(A) \leq \mu(B) \quad \text{(monotonicity)}
\]

(2.3)

Usually, Lebesgue measure and probability measure are special case of additive measure. An additive measure must be a monotone measure, but a monotone measure may not be an additive measure. In fact, additive measure are special case of monotone measure. Dempster (1967) pointed out that a monotone measure is called non-additive measure or fuzzy measure. Later, Shafer (1976) improved this viewpoint. Sugeno

Let $g$ be a fuzzy measure on $\left(X, 2^X\right)$ and there are four kinds of additive measure for $g$.

1. If $\forall A, B \in 2^X$, $A \cap B = \phi$, $\exists g(A \cup B) = g(A) + g(B)$, then $g$ is an additive measure on $\left(X, 2^X\right)$.

2. If $\forall A, B \in 2^X$, $A \neq \phi, B \neq \phi, A \cap B = \phi, A \cup B \neq X \exists g(A \cup B) > g(A) + g(B)$, then $g$ is a super-additive measure on $\left(X, 2^X\right)$.

3. If $\forall A, B \in 2^X$, $A \neq \phi, B \neq \phi, A \cap B = \phi, A \cup B \neq X \exists g(A \cup B) < g(A) + g(B)$, then $g$ is a sub-additive measure on $\left(X, 2^X\right)$.

4. If $g$ does not belong (1), (2), or (3), it is called mixture fuzzy measure.

### 2-2-2 Fuzzy density function

**Definition 2.2** fuzzy density function

A fuzzy density function (Choquet, 1953; Liu, Tu, Chen, & Weng, 2008; Sugeno, 1974; Wang & Klir, 1992; Wang & Klir, 2009; Zadeh, 1978) of a fuzzy measure $\mu$ on a finite set $X$ is a function $s : X \rightarrow [0, 1]$ satisfying:

$$s(x) = \mu(\{x\}), x \in X$$

(2.4)

$s(x)$ is called the density of singleton $x$.

### 2-2-3 $\lambda$-measure

**Definition 2.3**

For a given singleton measures $s$, $\lambda$-measure (Wang & Klir, 2009) $g_{\lambda}$, is a fuzzy measure on a finite set $X$, satisfying:

$$A, B \in 2^X, A \cap B = \phi, A \cup B \neq X$$
\[ \Rightarrow g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B) \quad (2.5) \]

\[ \prod_{i=1}^{n} \left[ 1 + \lambda s(x_i) \right] = \lambda + 1 > 0, s(x_i) = g_\lambda(\{x_i\}) \quad (2.6) \]

where, the real number \( \lambda \), is also called the determinate coefficient of \( \lambda \)-measure.

Note that once the fuzzy density function is known, we can obtain the value of \( \lambda \) uniquely by using the previous polynomial equation. In other words, \( \lambda \)-measure has a unique solution without closed form, and is a univalent fuzzy measure, and if \( \sum_{x \in X} s(x) = 1 \) then \( \lambda \)-measure is just the additive measure.

### 2-2-4 P-measure

**Definition 2.4**

For each given fuzzy density function, \( s(x), x \in X \), on a finite set \( X \), a P-measure \( g_P \), is a fuzzy measure on set \( X \), satisfying (Zadeh, 1978):

1. \( g_P(\emptyset) = 0, g_P(X) = 1 \quad (2.7) \)
2. \( \forall A \subseteq X \Rightarrow g_P(A) = \max_{x \in A} s(x) = \max_{x \in A} g_P(\{x\}) \quad (2.8) \)

Note that P-measure is also a univalent fuzzy measure with only one fuzzy measure solution.

### 2-3 Multivalent Fuzzy Measures

**2-3-1 Definition of multivalent fuzzy measure**

**Definition 2.5** A fuzzy measure is called a multivalent measure, if it has more than one fuzzy measure solution (Zadeh, 1978; Choquet, 1953).

**2-3-2 Comparison between two measures**

**Definition 2.6** \( \mu_1 \) -measure \( \leq \mu_2 \) -measure
For any given fuzzy density function \( s(x) \) on a finite set \( X \), if \( \mu_1 \) and \( \mu_2 \) are two fuzzy measures, satisfying \( g_{\mu_1}(A) \leq g_{\mu_2}(A) \), \( \forall A \subseteq X \), then we say that \( \mu_1 \)-measure is not larger than \( \mu_2 \)-measure, or \( \mu_2 \)-measure is not smaller than \( \mu_1 \)-measure (Zadeh, 1978; Choquet, 1953), denoted as
\[
\mu_1 \text{-measure} \leq \mu_2 \text{-measure}.
\] (2.9)

**Theorem 2.1** For any given fuzzy density function \( s(x) \) on a finite set \( X \), \( P \)-measure is not larger than any other fuzzy measure \( \mu \), that is\[
P \text{-measure} \leq \mu \text{-measure}
\] (2.10)

### 2-4 \( \delta \)-measure

Since \( L \)-measure does not include the additive measure, an improved multivalent fuzzy measure, called \( \delta \)-measure, was proposed by Liu’s previous work as following definition.

**Definition 2.7 \( \delta \)-measure**

For given singleton measures \( s(x) \), a \( \delta \)-measure is a multivalent fuzzy measure with determine coefficient \( \delta \in [-1,1] \) on a finite set \( X \), \( |X|=n \), satisfying (Liu, Wu, Jheng, & Sheu, 2009; Liu, Chen, Wu, & Sheu, 2009):

1. \[
\sum_{x \in X} s(x) = 1
\] (2.11)

2. \[
g_\delta(\emptyset) = 0, g_\delta (X) = 1, g_\delta (\{x\}) = s(x), \ \forall x \in X
\] (2.12)

3. \[
\forall A \subseteq X, 1 < |A| < |X| \Rightarrow g_\delta (A) = \begin{cases} \frac{\max_{x \in A} s(x)}{1 + \delta \max_{x \in A} s(x)} & \text{if } \delta = -1 \\ \frac{[1 + \delta \max_{x \in A} s(x)](1 + \delta \sum_{x \in A} s(x)) - \delta \max_{x \in A} s(x)}{1 + \delta \sum_{x \in A} s(x)} & \text{if } \delta \in (-1,1] \end{cases}
\] (2.13)
Theorem 2.2
For given singleton measure $s$,
If $A \subseteq B \subseteq X$ then
\[
\sum_{x \in B} s(x) - \sum_{x \in A} s(x) \geq \max_{x \in B} \{s(x)\} - \max_{x \in A} \{s(x)\} \geq 0
\] (2.14)

Theorem 2.3
For given singleton measure $s$, $\forall \delta \in [-1,1]$, $\delta$-measure is a fuzzy measure.

Theorem 2.4 Important properties of $\delta$-measure
(1) $\delta$-measure is an increasing and continuous function of $L$ on $[−1,1]$.
(2) $\forall \delta \in [-1,1]$, $\delta$-measure is a fuzzy measure, in other words, $\delta$-measure is a multivalent fuzzy measure with infinite many solutions.
(3) if $\delta = -1$ then $\delta$-measure is just the $P$-measure,
(4) if $\delta = 0$ then $\delta$-measure is just the additive measure,
(5) if $-1 \leq \delta < 0$ then $\delta$-measure is a sub-additive fuzzy measure,
(6) if $0 < \delta \leq 1$ then $\delta$-measure is a supper-additive fuzzy measure,

Theorem 2.5
If $\sum_{x \in X} s(x) = 1$ and $\delta = 0$ then $\delta$-measure is just the $\lambda$-measure

Theorem 2.6
$P$-measure, additive measure and $\lambda$-measure are the special cases of $\delta$-measure
2-5 Composed Measure of L-method and Delta-measure

2-5-1 L-measure

Definition 2.8 L-measure

For each given fuzzy density function $s$, a L-measure $g_L$, is a fuzzy measure on a finite set $X$, satisfying:

\begin{align}
(1) \quad g_L(\emptyset) &= 0, \quad g_L(X) = 1 \\
(2) \quad L \in [0, \infty), \quad \forall A \subseteq X, \quad A \neq X
\end{align}

\[\forall A \subseteq X, |X| - |A| + (|A| - 1)L > 0 \Rightarrow\]

\[g_L(A) = \max_{x \in A} [s(x)] + \frac{(|A| - 1)L \sum_{x \in A} s(x) \left[1 - \max_{x \in A} [s(x)]\right]}{|X| - |A| + (|A| - 1)L \sum_{x \in X} s(x)} \]

(2.17)

where the real number $L$ is also called the determinate coefficient of $L$-measure.

Theorem 2.7 Important properties of L-measure

(1) L-measure is an increasing and continuous function of $L$ on $[0, \infty)$.

(2) $\forall L \in [0, \infty)$, L-measure is a fuzzy measure.

(3) if $L=0$ then L-measure is just the $P$-measure.

(4) L-measure has infinite many solutions with closed form and is a multivalent fuzzy measure.

2-5-2 Definition of Generalized L-measure

Definition 2.9 Generalized L-measure

For given singleton measure $s(x)$, a generalized L-measure based on a fuzzy measure, $\mu$, $L_\mu$, is a fuzzy measure on a finite set $X$, $|X| = n$, satisfying:

\begin{align}
(1) \quad L \in [0, \infty)
\end{align}

(2.18)
(2) \( \forall A \subset X, n - |A| + (|A| - 1)L > 0 \Rightarrow \)

\[
g_{L,\rho}(A) = \max_{x \in A} \left[ s(x) \right] + \frac{(|A| - 1)L\mu(A) \left[ 1 - \max_{x \in A} \left[ s(x) \right] \right]}{n - |A| + (|A| - 1)L}\mu(X)
\]

(2.19)

Where the real number, \( L \), is also called the determine coefficient of \( L_{\mu} \)-measure.

**Theorem 2.8**

1. For each \( L \in [0, \infty) \), \( L_{\mu} \)-measure is a fuzzy measure, In other words, \( L_{\mu} \)-measure has infinite many fuzzy measures with determine coefficient \( L \), \( L \in [0, \infty) \).
2. \( L \in [0, \infty) \), \( L_{\mu} \)-measure is an increasing function on \( L \),
3. If \( L = 0 \) then \( L_{\mu} \)-measure is just the \( \mu \)-measure.
4. If \( \mu \)-measure is the \( P \)-measure then \( L_{\mu} \)-measure is the \( L \)-measure.
5. For each \( L \in [0, \infty) \),

\( P \)-measure \( \leq \) \( L \)-measure \( \leq \) \( L_{\mu} \)-measure.

Though \( \delta \)-measure includes the additive measure, but it has multiple measure solutions as \( L \)-measure, therefore, an improved multivalent fuzzy measure, the composed fuzzy measure of \( L \)-measure and \( \delta \)-measure, denoted \( L_{\delta} \)-measure was proposed by our previous work as follows (Liu, Chen, Wu, & Sheu, 2009).

**Definition 2.10** \( L_{\delta} \)-measure

For given singleton measure \( s(x) \), the composed measure of \( L \)-measure and \( \delta \)-measure, denoted \( L_{\delta} \)-measure as \( g_{L,\delta} \), is a multivalent fuzzy measure with determine coefficient \( L \in [-1, \infty) \) on a finite set \( X \), satisfying (Zadeh, 1978; Choquet, 1953):
(1) \( \sum_{x \in X} s(x) = 1 \) 

(2) \( g_s(\phi) = 0, g_s(X) = 1, g_s(\{x\}) = s(x), \forall x \in X \)

(3) \( \forall A \subset X, 1 < |A| < |X| \Rightarrow \)

\[
g_{\omega L}(A) = \begin{cases} 
\max_{x \in d} s(x) & \text{if } L = -1 \\
\frac{1 + L \max_{x \in d} s(x)}{1 + L \sum_{x \in d} s(x)} - L \max_{x \in d} s(x) & \text{if } L \in (-1,0) \\
\sum_{x \in d} s(x) + \frac{|\delta| - 1}{\sum_{x \in d} s(x)} \left[1 - \sum_{x \in d} s(x)\right] & \text{if } L \in [0, \infty) \\
\end{cases}
\]

**Theorem 2.9 Important Properties of \( L_\delta \)-measure**

(1) \( \forall L \in [-1, \infty), \) \( L_\delta \)-measure is a fuzzy measure, in other words, \( L_\delta \)-measure is a multivalent fuzzy measure with infinite solutions.

(2) \( L_\delta \)-measure is an increasing and continuous function of \( L \) on \([-1, \infty)\).

(3) if \( L = -1 \) then \( L_\delta \)-measure is just the P-measure.

(4) if \( L = 0 \) then \( L_\delta \)-measure is just the additive measure.

(5) if \( -1 \leq L < 0 \) then \( L_\delta \)-measure is a sub-additive fuzzy measure.

(6) if \( 0 < L < \infty \) then \( L_\delta \)-measure is a supper-additive fuzzy measure.

(7) If \( \sum_{x \in X} s(x) = 1 \) and \( L = 0 \) then \( L_\delta \)-measure is just the \( \lambda \)-measure.

(8) \( P \)-measure, additive measure and \( \lambda \)-measure are the special cases of \( L_\delta \)-measure.
2-6 Choquet Integral Regression Models

2-6-1 Choquet Integral

Definition 2.11 Choquet Integral

Let $\mu$ be a fuzzy measure on a finite set $X$. The Choquet integral of $f_i : X \rightarrow R_i$ with respect to $\mu$ for individual $i$ is denoted by

$$\int_{c} f_i d\mu = \sum_{j=1}^{n} f_i(x_{(i)}) \cdot \mu(A_{(i)}) , i = 1,2,\cdots,N$$

(2.23)

where $f_i(x_{(0)}) = 0$, $f_i(x_{(j)})$ indicates that the indices have been permuted so that

$$0 \leq f_i(x_{(1)}) \leq f_i(x_{(2)}) \leq \cdots \leq f_i(x_{(n)})$$

(2.24)

$$A_{(j)} = \{x_{(j)},x_{(j-1)},\cdots,x_{(n)}\}$$

(2.25)

2-6-2 Choquet Integral Regression Models

Definition 2.12 Choquet Integral Regression Models

Let $y_1,y_2,\cdots,y_N$ be global evaluations of $N$ objects and $f_1(x_j),f_2(x_j),\cdots,f_N(x_j)$, $j = 1,2,\cdots,n$, be their evaluations of $x_j$, where $f_i : X \rightarrow R_i$ , $i = 1,2,\cdots,N$.

Let $\mu$ be a fuzzy measure, $\alpha,\beta \in R$,

$$y_i = \alpha + \beta \int_{c} f_i dg_{\mu} + e_i , e_i \sim N(0,\sigma^2) , i = 1,2,\cdots,N$$

(2.26)

$$\left(\hat{\alpha},\hat{\beta}\right) = \arg\min_{\alpha,\beta} \left[ \sum_{i=1}^{N} \left( y_i - \alpha - \beta \int_{c} f_i dg_{\mu} \right)^2 \right]$$

(2.27)

then $\hat{y}_i = \hat{\alpha} + \hat{\beta} \int_{c} f_i dg_{\mu}$, $i = 1,2,\cdots,N$ is called the Choquet integral regression equation of $\mu$ (Liu, 2009; Liu, Chen, Jheng, & Chien, 2009; Liu, Chen, Wu, & Sheu, 2009; Liu, Tu, Chen, & Weng, 2008; Liu, Wu, Jheng, & Sheu, 2009), where

$$\hat{\beta} = \frac{S_{yf}}{S_{ff}}$$

(2.28)
\[ \hat{\alpha} = \frac{1}{N} \sum_{i=1}^{N} y_i - \frac{1}{N} \sum_{j=1}^{N} \left[ f_i d_{g_{\mu}} \right] \]  

\[ S_{yf} = \frac{\frac{1}{N} \sum_{i=1}^{N} y_i - \frac{1}{N} \sum_{j=1}^{N} y_i \left[ \int f_i d_{g_{\mu}} - \frac{1}{N} \sum_{k=1}^{N} \int f_k d_{g_{\mu}} \right]}{N-1} \]  

\[ S_{yf} = \frac{\frac{1}{N} \sum_{i=1}^{N} \left[ \int f_i d_{g_{\mu}} - \frac{1}{N} \sum_{k=1}^{N} \int f_k d_{g_{\mu}} \right]^2}{N-1} \]  

2-7 Fuzzy Density Function

**Definition 2.13** For given singleton measure \( s \) of a fuzzy measure \( \mu \) on a finite set \( X \), if \( \sum_{x \in X} d(x) = 1 \), then \( s \) is called a density function of \( \mu \).

**Definition 2.14** \( \gamma \)-density function

Let \( \mu \) be a fuzzy measure on a finite set \( X = \{x_1, x_2, ..., x_n\} \), \( y_i \) be global response of subject \( i \) and \( f_i(x_j) \) be the evaluation of subject \( i \) for singleton \( x_j \), satisfying:

\[ 0 < f_i(x_j) < 1, \quad i = 1, 2, ..., N, \quad j = 1, 2, ..., n \]

\[ \gamma(x_j) = \frac{1 + r(f(x_j))}{\sum_{k=1}^{N} [1 + r(f(x_k))]}, \quad j = 1, 2, ..., n \]  

Where

\[ r(f(x_j)) = \frac{S_{y,y_j}}{S_y S_{y_j}} \]  

\[ S_{y_j}^2 = \frac{1}{N} \sum_{i=1}^{N} \left( y_i - \frac{1}{N} \sum_{j=1}^{N} y_i \right)^2 \]  

\[ S_{y_j}^2 = \frac{1}{N} \sum_{i=1}^{N} \left[ f_i(x_j) - \frac{1}{N} \sum_{j=1}^{N} f_i(x_j) \right]^2 \]  

\[ S_{y,y_j} = \frac{1}{N} \sum_{i=1}^{N} \left( y_i - \frac{1}{N} \sum_{j=1}^{N} y_i \right) \left[ f_i(x_j) - \frac{1}{N} \sum_{j=1}^{N} f_i(x_j) \right] \]
satisfying \( 0 \leq \gamma(x_j) \leq 1 \) and \( \sum_{j=1}^{n} \gamma(x_j) = 1 \) \( (2.37) \)

then the function \( \gamma : X \to [0,1] \) satisfying \( \mu(\{x\}) = \gamma(x), \ \forall x \in X \) is a fuzzy density function of \( \mu \), called \( \gamma \)-density function of \( \mu \).
3-1 Complete $L$-measure

For each given fuzzy density function, $L$-measure is a multivalent fuzzy measure with infinite many fuzzy measure solutions including the smallest fuzzy measure, $P$-measure, but it can not attain to the largest fuzzy measure, $B$-measure, and is not a completed fuzzy measure, $B$-measure, and is not a completed fuzzy measure. $B$-measure and completed $L$-measure was proposed by Liu (Liu, 2009; Liu, Wu, Chen, & Jheng, 2010), their formal definitions are described as follows.

3-1-1 B-measure

**Definition 3.1** B-measure

For any given fuzzy density function $s(x)$ on a finite set $X$, a B-measure is a set function $g_B : 2^X \to [0,1]$, satisfying (Liu, 2009):

$$g_B(A) = \begin{cases} 0 & A = \emptyset \\ s(x) & A = \{x\}, x \in X \\ 1 & |A| > 1, A \subset X \end{cases}$$ (3.1)

**Theorem 3.1** for any given fuzzy density function $s(x)$ on a finite set $X$, B-measure is not smaller than any fuzzy measure $\mu$, that is

$$B \text{-measure} \geq \mu \text{-measure}$$.

In other words, for any given fuzzy density function $s(x)$ on a finite set $X$, B-measure is the largest fuzzy measure.

3-1-2 Completed $L$-measure

Completed fuzzy measure and Completed $L$-measure were also proposed by Hsiang-Chuan Liu in 2009 (Liu, 2009; Liu, Wu, Chen, & Jheng, 2010). Its formal definition is described as follows;
Definition 3.2  Completed measure

For any given fuzzy density function \( s(x) \) on a finite set \( X \), a multivalent fuzzy measure \( \mu \)-measure with determinate coefficient \( \mu \) is called a completed measure, if it satisfies following conditions (Liu, 2009; Liu, Wu, Chen, & Jheng, 2010):

1. If \( \mu = 0 \) then \( \mu \)-measure is just the P-measure.
2. If the upper limit fuzzy measure of \( \mu \)-measure is just the B-measure.

Note that \( L \)-measure is not a completed measure, since

\[
\lim_{L \to \infty} g_L(A) \neq g_\infty(A)
\]  \hspace{3cm} (3.2)

Definition 3.3  Completed \( L \)-measure, \( L_C \)-measure (Liu, 2009; Liu, Wu, Chen, & Jheng, 2010)

For any given fuzzy density function \( s(x) \) on a finite set \( X \), a Completed \( L \)-measure, \( L_C \)-measure, is a set function \( g_{L_C} : 2^X \to [0,1] \) satisfying:

1. \( g_{L_C}(\emptyset) = 0, \ g_{L_C}(X) = 1 \)
2. \( L \in [0,\infty), \forall A \subset X, A \neq X \)

\[
g_{L_C}(A) = \max_{x \in A} \{s(x)\} + \frac{(|A| - 1)L \sum_{x \in A} s(x) \left[1 - \max_{x \in A} \{s(x)\}\right]}{|X| - |A| \sum_{x \in X} s(x) + (|A| - 1)L \sum_{x \in A} s(x)}
\]  \hspace{3cm} (3.3)

Theorem 3.2  The properties of \( L_C \)-measure (Liu, 2009; Liu, Wu, Chen, & Jheng, 2010)

1. \( L_C \)-measure is an increasing and continuous function of \( L \) on \( [0,\infty) \).
2. \( \forall L \in [0,\infty), \ L_C \)-measure is a fuzzy measure.
3. if \( L = 0 \) then \( L_C \)-measure is the \( P \)-measure.
4. if \( L \to \infty \) then \( L_C \)-measure is the \( B \)-measure.
5. \( L_C \)-measure has infinite many solutions with closed form and is a multivalent
fuzzy measure.

(6) $L_{c \delta}$-measure is a completed fuzzy measure, satisfying

$$\lim_{L \to \infty} g_{Lc \delta}(A) = g_{\delta}(A)$$

3-2 Composed fuzzy measure of completed $L$-measure and $\delta$-measure

3-2-1 Definition of the composed fuzzy measure of completed $L$-measure and $\delta$-measure

**Definition 3.4** $L_{c \delta}$-measure (Liu, 2009; Liu, Wu, Chen, & Jheng, 2010)

For given singleton measure $s(x)$, the composed measure of completed $L$-measure and $\delta$-measure, denoted $L_{c \delta}$-measure, $g_{L_{c \delta}}$ is a multivalent fuzzy measure with determine coefficient $L \in [-1, \infty)$ on a finite set $X$, satisfying:

1. $\sum_{x \in X} s(x) = 1$ (3.4)
2. $g_{\delta}(\emptyset) = 0, g_{\delta}(X) = 1, g_{\delta}(\{x\}) = s(x), \forall x \in X$ (3.5)
3. $\forall A \subset X, 1 < |A| < |X| \Rightarrow$

$$g_{L_{c \delta}}(A) = \begin{cases} \max_{x \in A} s(x) & \text{if } L = -1 \\ \frac{1 + \max_{x \in A} s(x)}{1 + \sum_{x \in A} s(x)} - L \sum_{x \in A} s(x) & \text{if } L \in (-1, 0) \\ \sum_{x \in A} s(x) + \frac{(|A| - 1) \sum_{x \in A} s(x) - \sum_{x \in A} s(x)}{(|X| - |A|) \sum_{x \in A} s(x) + (|A| - 1) \sum_{x \in A} s(x)} & \text{if } L \in [0, \infty) \end{cases}$$ (3.6)

3-2-2 Important properties of $L_{c \delta}$-measure (Liu, 2009; Liu, Wu, Chen, & Jheng, 2010)
Theorem 3.3  Important properties of $L_{C\delta}$-measure

(1) $L_{C\delta}$-measure is an increasing and continuous function of L on $[-1, \infty)$

(2) $\forall L \in [-1, \infty)$, $L_{C\delta}$-measure is a fuzzy measure, in other words, $L_{C\delta}$-measure is a multivalent fuzzy measure with infinite many solutions.

(3) if $L = -1$ then $L_{C\delta}$-measure is just the $P$-measure,

(4) if $L = 0$ then $L_{C\delta}$-measure is just the additive measure,

(5) if $L = 0$ and $\sum_{x \in X} s(x) = 1$ then $L_{C\delta}$-measure is just the $\lambda$-measure,

(6) if $-1 \leq L < 0$ then $L_{C\delta}$-measure is a sub-additive fuzzy measure,

(7) if $0 < L < \infty$ then $L_{C\delta}$-measure is a super-additive fuzzy measure,

(8) $L_{C\delta}$-measure is a completed fuzzy measure, satisfying

$$\forall A \subset X, |A| > 1 \Rightarrow \lim_{L \to \infty} L_{C\delta}(A) = L_\delta(A). \quad (3.7)$$

Proof.

(1) if $L \in [-1, 0)$, then $L_{C\delta}$-measure is a special case of $\delta$-measure, since $\delta$-measure is a fuzzy measure, then $L_{C\delta}$-measure is also a fuzzy measure.

if $L \in [0, \infty)$, then $L_{C\delta}$-measure is a special case of generalized $L$-measure based on the additive measure, since any generalized $L$-measure is also a fuzzy measure, then $L_{C\delta}$-measure is also a fuzzy measure.

Therefore, for each $L \in [-1, \infty)$, $L_{C\delta}$-measure is a fuzzy measure.

(2) if $L \in [-1, 0)$, then $L_{C\delta}$-measure is a special case of $\delta$-measure, since $\delta$-measure is an increasing function with upper bound, additive measure, then $L_{C\delta}$-measure is also an increasing function with upper bound, additive measure.

if $L \in [0, \infty)$, then $L_{C\delta}$-measure is a special case of generalized $L$-measure based on the additive measure, since generalized $L$-measure based on the additive measure is also an increasing function with lower bound, additive measure, then $L_{C\delta}$-measure is also an increasing function with lower bound,
additive measure.
Therefore, for each $L \in [-1, \infty)$, $L_{c0}$-measure is also an increasing function on L. (3), (4), (5), (6), (7) and (8) are trivial.
CHAPTER IV  THE EXPERIMENT AND RESULTS

This chapter first presents an analysis and interpretation of the data used in this study. Then an analysis and comparison of the data related to the many fuzzy measures is presented.

4-1 Data of the Basic Competence Test

The total scores of 60 students from a junior high school in Taiwan are used for this research (Liu, Chen, Wu, & Sheu, 2009; Liu, Wu, Jheng, & Sheu, 2009). The examinations of four courses, physics and chemistry, biology, geoscience and mathematics, are used as independent variables, the score of the Basic Competence Test of junior high school is used as a dependent variable.

The data of all variables listed in Table II which was applied to evaluate the performances of seven Choquet integral regression models with $P$-measure, $\lambda$-measure, $\delta$-measure, $L$-measure, $L_\delta$-measure, $L_C$-measure, and $L_{C\delta}$-measure based on $\gamma$-density function respectively, a ridge regression model, and a multiple linear regression model by using 5-fold cross validation method to compute the mean square error (MSE) of the dependent variable. The formula of MSE is

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \quad (4.1)$$

The same singleton measure set, $\gamma$-density function, of the aforementioned fuzzy measures is listed as follows which can be obtained by using the formula (2.32).

\{0.2229, 0.2848, 0.2567, 0.2356\} \quad (4.2)

4-2 Experimental Results

For any fuzzy measure, $\mu$-measure, once the fuzzy density function of the $\mu$-measure is given, all event measures of $\mu$ can be found, and then, the Choquet
integral based on $\mu$ and the Choquet integral regression equation based on $\mu$ can also be found by using above corresponding formulae.

**TABLE I. MSE OF REGRESSION MODELS**

<table>
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<th>Regression model</th>
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<td>Multiple linear regression</td>
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The experimental results of nine forecasting models are listed in Table I. It shows that the Choquet integral regression model with $L_{cs}$-measure based on $\gamma$-density function outperforms other forecasting regression models.
CHAPTER V  CONCLUSIONS AND RECOMMENDATIONS

This research sought to find out the effect of $L_{C\delta}$-measure.

5-1  Conclusions

The following conclusions were drawn from this study:

1. A multivalent composed fuzzy measure of completed L-measure and $\delta$-measure, denoted $L_{C\delta}$-measure, is proposed. This new measure is proved that it is of closed form with infinitely many solutions, and it can be considered as an extension of the three well known measures: additive measure, $\lambda$-measure, and $P$-measure, respectively.

2. This improved multivalent fuzzy measure is a continuous and increasing function of L on $[-1, \infty)$. It not only includes the smallest fuzzy measure, $P$-measure, but also attains to the largest fuzzy measure. It is a completed multivalent fuzzy measure and has more range of infinitely many fuzzy measure solutions than which of $L_{\delta}$-measure.

3. By using 5-fold cross-validation MSE, a real data experiment is conducted for comparing the performances of a multiple linear regression model, a ridge regression model, and the Choquet integral regression model with respect to $P$-measure, $\lambda$-measure, $\delta$-measure, L-measure, $L_{\delta}$-measure and the new fuzzy measure, $L_{C\delta}$-measure based on $\gamma$-density function respectively. The result shows that the Choquet integral regression models with respect to the proposed $L_{C\delta}$-measure based on $\gamma$-density function outperforms other forecasting models.

5-2  Suggestion

The recommendations which are put forth in this section have been drawn from
experience in this study and from an analysis of its results.

1. From this study, the result shows that the Choquet integral regression models with respect to the proposed $L_{C;\delta}$ -measure based on $\gamma$-density function outperforms other forecasting models. It strongly recommends applying the $L_{C;\delta}$ -measure in decision making, commercial, credit risk modeling, banking industry and educations.

2. It recommends to use simulation to validate the Choquet integral regression models with respect to the proposed $L_{C;\delta}$ -measure based on $\gamma$-density function outperforms other forecasting models.

3. For further study, researchers might consider to generalize the $L_{C;\delta}$ -measure to signed measure as their research interests.
REFERENCES


Appendix

Appendix A: Table II

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C1 : physics and chemistry  
C2 : biology  
C3 : geoscience  
C4 : mathematics  
BCT : Basic Competence Test of nature science
Appendix B: Published Paper I


Appendix C: Published Paper II


Appendix D: Published Paper III

VITA

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BIRTHDATE: July, 1958

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EDUCATION:

Doctor of Philosophy, 2010
Graduate Institute of Educational Measurement and Statistics,
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Doctor of Philosophy, 1994
Department of Mathematics
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M.S., 1985
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B.S., 1983
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Taichung Junior Teachers College
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EXPERIENCE:

1994-present Associate Professor,
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1988-1994 Instructor,
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1985-1988 Teaching Assistant
National Taichung Teachers College, Taiwan

1979-1983 Teacher
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PUBLISHING (Under 5 years)
A. Journal Papers

http://www.oldcitypublishing.com/MVLSC/MVLSC.html


B. Conference Papers


Wu, D. B. & Ma, H. L. (2010). An application of GM(0,n) to analyze the ma-wu’s test of Practical reasoning abilities. *Proceedings of the Sixth IASTED International Conference Advances in Computer Science and Engineering (ACSE 2010)*


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